Jakarta International School

7th Grade

Name: ANSWER KEY

Date: _____

Practice Test - BLACK

Operations with Fractions

Score: 30/30

Clearly SHOW or EXPLAIN how you arrive at ALL your answers !!!

1. A college class has exactly enough students to form eight equal rows. On Monday, a student is absent, and the professor is able to seat the students into five equal rows. On Tuesday, two students are absent, and the professor can seat the students into nine equal rows. What is the least possible number of students in the class? **(2 pts)**

Let x be the number of students in the college class. We know x is a multiple of 8 since the students fit in eight equal rows. The fact that the class fits in five equal rows when one student is absent suggests that x is one more than a multiple of 5. The least number that meets these two conditions is 16. The next number to meet these two conditions would have to be 40 more than 16 since 40 is the least common multiple of 8 and 5. In general, we know from the first two restrictions that $x = 16 + 40k$ where k is a non-negative integer. Now we consider the third condition. When two students are absent, the class fits in 9 rows. This means that x is 2 more than a multiple of 9. Let's look at numbers of the form $16 + 40k$ for a number that is two more than a multiple of 9. When $k = 0$, we get $16 + 0 = 16$, which is 7 more than a multiple of 9. When $k = 1$, we get $16 + 40 = 56$, which is 2 more than 6×9. The least possible number of students in the class is **56**.

2. Using the following three clues, can you figure out which integer I am? (1) If I am not a multiple of 4, then I am between 60 and 69. (2) If I am a multiple of 3, I am between 50 and 59. (3) If I am not a multiple of 6, I am between 70 and 79. What integer am I? **(2 pts)**

Let's assume our mystery number is a multiple of 3, so it must be either 51, 54 or 57 according to the second statement. However, it can not be 51, 54 or 57 because then it would not be a multiple of 4, and if it is not a multiple of 4, according to the first statement it must be between 60 and 69. Now we know the mystery number is not a multiple of 3. Because it is not a multiple of 3, it certainly can't be a multiple of 6 either, so it must be either 71, 73, 74, 75, 76, 77 or 79, according to the third statement. This means the mystery number is a multiple of 4, since if it is not a multiple of 4, it would have to be a number between 60 and 69. Of the numbers 71, 73, 74, 75, 76, 77 and 79, only 76 is a multiple of 4. Our mystery number is **76**.

3. What is the sum of the integers K such that $\frac{k}{21}$ is greater than $-\frac{2}{3}$ and less than $-\frac{2}{7}$? **(2 pts)**

For that to be true,

K can be $-13, -12, -11, -10, -9, -8, -7$

$\frac{-14}{21} < \frac{K}{21} < \frac{-6}{21}$

Sum = -70

4. Let a and b represent nonzero integers. Find a rational number of the form $\frac{a}{b}$ so that $2.\overline{3} < \frac{a}{b}$ and $\frac{a}{b} < 2\frac{7}{20}$. (2 pts)

$2.\overline{3} = 2\frac{1}{3} = 2\frac{20}{60}$

$2\frac{7}{20} = 2\frac{21}{60}$

So, we want $2\frac{20}{60} < \frac{a}{b} < 2\frac{21}{60}$

$\frac{140}{60} < \frac{a}{b} < \frac{141}{60}$

$\frac{280}{120} < \frac{a}{b} < \frac{282}{120}$

So, $\frac{a}{b}$ could $= \frac{281}{120}$

5. Write the decimal $0.3\overline{21}$ as a fraction. (2 pts)

Let $x = .3\overline{21}$

$100x = 32.1\overline{21}$

$-\ x = .3\overline{21}$

$99x = 31.8$

$\frac{990x}{990} = \frac{318}{990}$

$x = \frac{318 \div 2}{990 \div 2}$

$x = \frac{159 \div 3}{495 \div 3}$

$x = \frac{53}{165}$

6. Find the indicated square root: $\sqrt{1024} =$ (2 pts)

$1024 = 512 \cdot 2 = 256 \cdot 4 = 128 \cdot 8 = 64 \cdot 16 = 32 \cdot 32$

So $\sqrt{1024} = 32$

7. Use the divide and average method to find an approximation for the square root (to the nearest hundredth): $\sqrt{250}$ (2 pts)

$\sqrt{250}$ lies between 15^2 and 16^2

250 is closer to 256, so let's guess 15.6

$250 \div 15.6 \approx 16.03$

Average of 16.03 and $15.6 \approx 15.82$

$250 \div 15.82 \approx 15.80$

Since 15.82 and 15.80 only agree to tenths place, try 15.81.

$250 \div 15.81 \approx 15.81$

So $\sqrt{250} = 15.81$

8. **Explain**. What is the difference between a rational and irrational number? Give an example of each *using radical symbols.* (2 pts)

Rational numbers can be represented as fractions. Consequently, they result in repeating or non-repeating decimals. Eg. $\sqrt{16} = \frac{4}{1}$ and $\sqrt{\frac{1}{9}} = \frac{1}{3} = .\overline{3}$ are rational numbers. Irrational numbers have no equivalent fractions. Therefore, their decimals are neither terminating nor repeating. Eg. $\sqrt{2}$ cannot be written as a fraction. $\sqrt{2} = 1.41421356\ldots$

9. Kevin spent $\frac{4}{25}$ of his money on a magazine, $\frac{3}{10}$ of it on a book, $\frac{2}{3}$ of the remainder on a dictionary. If he spent \$3 more on the dictionary than on the book, how much money did he have at first? (2 pts)

Let x = total money spent.

Money spent on:

Magazine and Book $= \frac{4}{25}x + \frac{3}{10}x = \frac{23}{50}x$

Money remaining $= \frac{27}{50}x$

Money spent on dictionary $= \frac{2}{3} \cdot \frac{27}{50}x = \frac{9}{25}x$

Book $= \frac{3}{10}x$

Kevin spent \$3 more on the dictionary than the book, so

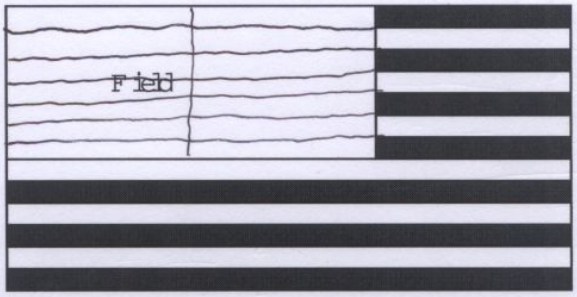
$\frac{9}{25}x - 3 = \frac{3}{10}x$

$50 \cdot \frac{9}{25}x - 50 \cdot 3 = 50 \cdot \frac{3}{10}x$

$18x - 150 = 15x$

$x = \$50$

10. The length of the field on the flag diagrammed below is 2/3 the length of the entire flag (drawing not exactly to scale). There are seven dark stripes and 6 white stripes. Though the stripes differ in length, they are of equal width. What fractional part of the flag's area is represented by the shaded stripes? Express your answer as a common, simplified fraction. (2 pts)



Since the short stripes are $\frac{1}{3}$ the length of the longer stripes, we know that 3 short stripes equals 1 long stripe.

So, altogether, there are

$4\frac{1}{3}$ black stripes, 4 white stripes, and $\frac{14}{3} = 4\frac{2}{3}$ "Field" stripes.

So, total stripes $= 4\frac{1}{3} + 4 + 4\frac{2}{3} = 13.$

Fraction shaded is $\frac{4\frac{1}{3}}{13} = 4\frac{1}{3} \div 13$

$= \frac{13}{3} \cdot \frac{1}{13} = \boxed{\frac{1}{3}}$

11. Calculate the sum of the geometric series $1+\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)^3+\ldots$ Express your answer as a common fraction. (2 pts)

Let $x = 1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 \ldots$

then $\frac{1}{4}x = \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \ldots$

So $x = 1 + \frac{1}{4}x$

$\frac{3}{4}x = 1$

so $\boxed{x = \frac{4}{3}}$

12. If $\frac{2}{3}$ of a ton of hay costs $\frac{4}{5}$ of an eagle, how many eagles will $\frac{1}{6}$ of a ton cost? What is the number of eagles in the answer to this problem? Express your answer as a common fraction. (2 pts)

Algebraically, we are given that $\frac{2}{3}h = \frac{4}{5}e$. Because $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$, we can solve this immediately:

$$\frac{1}{4} \times \frac{2}{3}h = \frac{1}{4} \times \frac{4}{5}e$$
$$\frac{1}{6}h = \frac{1}{5}e$$

Hence, $\frac{1}{6}$ of a ton will cost $\boxed{\frac{1}{5} \text{ eagles.}}$

13. If b is negative, what is the value of b in the geometric sequence 25, a, 56.25, b? Express your answer as a common fraction. (2 pts)

To find the next terms in a geometric sequence, you always multiply by the same factor. Moving from 25 to a to 56.25, we must multiply by the same factor twice. Call the factor x. So $25x^2 = 56.25$

$x^2 = 56\frac{1}{4} \div 25$

$\frac{225}{4} \cdot \frac{1}{25} = \frac{225}{100} = \frac{9}{4}$

so $x = \frac{3}{2}$

$b = 56\frac{1}{4} \cdot \frac{3}{2}$

$b = \frac{225}{4} \cdot \frac{3}{2} = \frac{675}{8}$

$\boxed{b = 84\frac{3}{8}}$

14. Simplify: $\frac{x^2-2x-3}{x+1} \bullet 5(x-3)^{-1}$ (2 pts)

$\frac{(x-3)(x+1)}{(x+1)} \cdot 5 \cdot \frac{1}{(x-3)} = \boxed{5}$

15. David bought $\frac{2}{5}$ kg of vegetables, $\frac{3}{8}$ kg of prawns and $\frac{1}{4}$ kg of meat for \$13.20 altogether. He bought the meat for \$5 per kg. If he had not bought the veggies, but another $\frac{3}{8}$ kg of prawns instead, he would have spent about \$2.15 more altogether. How much did he pay for each kg of veggies? (2 pts)

Let v = price of veggies per kilogram
p = price of prawns per kilogram

$\frac{2}{5}v + \frac{3}{8}p + \$5\left(\frac{1}{4}\right) = \$13.20$

$\frac{2}{5}v + \frac{3}{8}p = \$11.95$

From the 3rd sentence,

$0v + \left(\frac{3}{8} + \frac{3}{8}\right)p = \$14.10$

$\frac{3}{4}p = 14.10$

$p = \$18.80$

$\frac{2}{5}v + \frac{3}{8} \cdot 18.80 = 11.95$

$\boxed{v = \$12.25 \text{ per kilogram}}$