Jakarta International School

7th Grade

Practice Test - BLACK

Factors, Fractions, and Exponents

Name: SOLUTIONS

Date: ____________

Score: ___ / 23

Clearly SHOW or EXPLAIN how you arrive at ALL your answers !!!

1. Divisible by a Dozen

12

Take any three consecutive integers. Multiply the first times the square of the second, then multiply by the third.

Example:

If you use 2, 3, 4 then you would have $2 * 3^2 * 4$, which equals 72.

A. Try this with four different groups of three consecutive integers. Is each result divisible by 12? (1 point)

$3 \cdot 4^2 \cdot 5 = 240$ $\quad 4 \cdot 5^2 \cdot 6 = 600$ $\quad$ YES, each result is divisible by 12.

$1 \cdot 2^2 \cdot 3 = 12$

B. Show or explain why this is always true no matter what three integers you start with. (1 point)

The result will always have $2 \cdot 2 \cdot 3$ (12) in its prime factorization. If the middle number is even, squaring it will guarantee that $2 \cdot 2$ is part of its prime factorization. If the first and last numbers are even, each will contribute a prime factor of 2. There will always be a prime factor of 3 because there are 3 numbers and every 3rd number is a multiple of 3.

2. For what value of n is the five-digit number 7n,933 divisible by 33? To do this problem you need to know the divisibility rule for 11. The **divisibility rule for 11** is: A number is divisible by 11 if the difference between the sum of the odd numbered digits (1st, 3rd, 5th...) and the sum of the even numbered digits (2nd, 4th...) is 0 or is divisible by 11

(1 point)

$\boxed{n = 5}$

Divisibility by 33 requires that a number be divisible by both 11 and 3. If a five-digit number is divisible by 11, the difference between the sum of the units, hundreds and ten-thousands digits and the sum of the tens and thousands digits must be divisible by 11. Thus $(7+9+3)-(n+3) = 16-n$ must be divisible by 11. The only digit which can replace n for the number to be divisible by 11, then, is $n = 5$. Furthermore, if a number is divisible by 3, then the sum of its digits must also be divisible by 3. When $n = 5$, the sum of the digits of the number is $7+5+9+3+3 = 27$, so the number is divisible by 3. Hence, $n = 5$.

3. Find a way of writing 1,000,000 as a product of two numbers, neither of which ends in zero. (1 point) Use a factor tree to find the prime factorization of 1,000,000

$= 5^6 \cdot 2^6$

$= \boxed{15625 \cdot 64}$

4. List all numbers less than 100 with exactly 5 factors. Find the next two numbers, each greater than 100, that have exactly five factors. (1 point)

If a number has 5 factors, it must be of the form p^4 where p is a prime number. So, $2^4 = 16$ and $3^4 = 81$ are the only numbers less than 100 that have 5 factors. The next numbers would be $5^4 = 625$ and $7^4 = 2401$.

5. What is the least positive integer that has each of the first eight positive integers as factors? (1 point) $8! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40{,}320$. Our number does not need all seven factors of 2 in 8! The most it will need is three factors of 2 to be divisible by the 8. A number that is divisible by 8 certainly is divisible by 2 and 4, and the factor of 2 in the 6 also will be covered. Our number will only need one factor of 3, one of 5, and one of 7. So, our number is $2^3 \cdot 3 \cdot 5 \cdot 7 = \boxed{840}$

6. If $x \oplus y = (x^y)^x$, what is the units digit of $7 \oplus 5$? (1 point)

We are looking for the units digit of $(7^5)^7 = 7^{35}$.

The units digits of ~~p~~ powers of 7 occur in a cycle of 4: 7, 9, 3, 1, 7, 9, 3, 1, ...

The units digit of $7^{35} = 7^{4(8)+3}$ will be the same as the units digit of 7^3, or $\boxed{3}$

7. The five-digit whole number 3a,7b1 is a perfect square. What is the greatest possible value for the product ab? (1 point)

A perfect square whose unit digit is 1 must be the square of a number whose unit digit is either 1 or 9. $200^2 = 40{,}000$, so we can start with 199^2 and work down, looking for a 7 in the hundreds place.

$189^2 = 35{,}721$ is a candidate, with $a = 5$, $b = 2$, and the product $ab = 10$.

$181^2 = 32{,}761$ is the only other candidate, with $a = 2$, $b = 6$ and the product $ab = 12$

So, the greatest possible value for $a \cdot b = \boxed{12}$

8. What is x, if $x^{12} = 2$? Express your answer as a decimal to the nearest hundredth. (1 point)

If $x^{12} = 2$, then $x = \sqrt[12]{2}$, which we read as "the twelfth root of 2." This is the same as $2^{\frac{1}{12}}$ ("two to the one-twelfth power") which we can enter into the calculator using the "y^x" key.

We could also raise some numbers to the 12th power until we get 2. $1.1^{12} = 3.138$; $1.05^{12} = 1.796$, and $1.06^{12} = 2.012$.

The approximate value $\boxed{1.06}$ is good to the nearest hundredth.

9. John's Cousins:

"This might interest you, professor," said John. "My age and the ages of each of my three distant cousins are all prime numbers, and the sum of our ages is 50.

"In that case," said the professor, who knew John's age, "I can tell you the ages of your three cousins."

You do not share the professor's advantage of knowing John's age to start with, but nevertheless, can you tell the ages of his cousins? (note that the number 1 is not considered to be a prime). (1 point)

John is 43 and his cousins are 2, 2, and 3. For any other value of John's age, more than one combination would have existed, and the professor would not have had sufficient information to determine the cousin's ages.

10. A digital, 12 hour clock shows hours and minutes. During what fraction of the day will the clock show the digit 1 in its display? Express your answer as a common fraction. (1 point)

Starting at midnight, list the hours that show a 1 in the display: 12, 1, 10, 11. For these four hours, the digit 1 shows in the display for the entire 60 minutes. Next, list the minutes that include the digit 1: 01, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 31, 41, 51. For these 15 minutes, the digit 1 shows for 1 minute. Thus, the digit 1 appears in the minutes part of the display for 15 minutes out of each hour. Thus, in a 12-hour period, the digit 1 appears during the whole hour for 4 hours, and during 15 minutes for the other 8 hours, for a total of 6 hours. In a 24-hour period, the digit 1 appears for $2 \times 6 = 12$ hours, or $\frac{12}{24} = \frac{1}{2}$ the time.

11. Jill gives half her cards to Bill. Bill then gives half of his cards to Jill. Jill and Bill repeat this two-step process twice more. If Jill now has 37 cards and Bill has 19 cards, with how many cards did Bill start? (1 point) At the 6th stage, Jill has 37 cards and Bill has 19 cards. Bill was the last to give half his cards to Jill, so he must have had $2 \times 19 = 38$ cards at the fifth stage, and Jill must have had $37 - 19 = 18$ cards. At the 4th stage, Jill must have had $2 \times 18 = 36$ cards, and Bill must have had $38 - 18 = 20$ cards. At the third stage, Bill must have had $2 \times 20 = 40$ cards, and Jill had $36 - 20 = 16$ cards. Jill must have had $2 \times 16 = 32$ cards at the second stage and Bill had $40 - 16 = 24$ cards. At the first stage, Bill must have had $2 \times 24 = 48$ cards, and Jill had $32 - 24 = 8$ cards. At the zero stage, Jill must have had $2 \times 8 = 16$ cards, and Bill must have had $48 - 8 = \boxed{40 \text{ cards.}}$

12. Express the reciprocal of 3.8 as a common fraction. (1 point)

$$3.8 = 3\frac{8}{10} = \frac{38}{10}$$

Reciprocal of $\frac{38}{10}$ is $\frac{10}{38} = \frac{5}{19}$

13. Solve for x. Provide work or an explanation that makes the thinking that leads to your answer clear. (2 points)

A. $30^{17} = 6^x 5^x$

Since 6 and 5 are factors of 30, $30^{17} = 6^{17} \times 5^{17}$

$\boxed{x = 17}$

B. $3^3 \bullet 9^3 \bullet 27^3 \bullet 81^3 \bullet\bullet\bullet\bullet\bullet 2187^3 = 3^x$

$3^3 \cdot (3^2)^3 \cdot (3^3)^3 \cdot (3^4)^3 \ldots (3^7)^3$

$3^3 \cdot 3^6 \cdot 3^9 \cdot 3^{12} \cdot 3^{15} \cdot 3^{18} \cdot 3^{21}$

sum of exponents = 84

$\boxed{x = 84}$

14. Evaluate $(7^3+7^3+7^3+7^3+7^3+7^3+7^3)^{\frac{1}{2}} =$ (1 point)

$7^1(7^3)^{\frac{1}{2}} \quad (7^4)^{\frac{1}{2}} = 7^2 = \boxed{49}$

15. One trillion is 10^n divided by one-millionth. What is the value of n? (1 point)

$10^{12} = \dfrac{10^n}{10^{-6}}$

$n - -6 = 12$

$n + 6 = 12$

$\boxed{n = 6}$

16. Ocean Water Molecules:

A. According to the World Book Encyclopedia, there are about 326 million cubic miles of water on the earth. One cubic mile is 5280^3 cubic feet. About how many cubic feet of water are there on the earth? Express your answer in scientific notation. (1 point)

$3.26 \times 10^8 \times 1.47 \times 10^{11}$

4.79×10^{19} cubic feet.

B. A thimble the size of your thumb holds about 5×10^{-4} cubic foot of water. How many thimblefuls of water are there on the earth? (1 point)

$5 \times 10^{-4} \sim \quad \dfrac{4.79 \times 10^{19}}{5 \times 10^{-4}} = 0.958 \times 10^{23}$

or $\boxed{9.58 \times 10^{22}}$

C. A cubic foot of water has about 9.47×10^{26} water molecules. Which is greater, the number of water molecules in a thimbleful of water or the number of thimblefuls of water on the earth? (1 point)

$(5 \times 10^{-4})(9.47 \times 10^{26}) = 47.35 \times 10^{22}$ or 4.735×10^{23}

The number of H_2O molecules in a thimbleful of water is great

D. How many times as great? (1 point)

$\dfrac{4.735 \times 10^{23}}{9.58 \times 10^{22}}$ 0.49×10^1 or 4.9 times as great

17. Santa's Little Helpers

With only a few days left until Christmas Eve, Santa's elves took a little break from their toy-making routine. A group of mischievous elves hid several sacks of toys. Elias Elf suggested that they create a puzzle for Santa to solve before they would return the toys. Here is the puzzle he suggested.

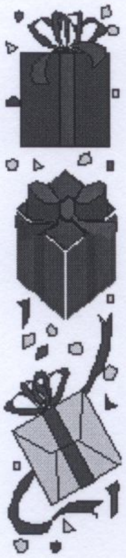
How many different ways can you trace the letters in the picture to spell out the word "TOYS"?

For example, you might trace

You could also trace

You cannot jump over letters, so you are not allowed to trace

Bobby Elf thought that this was a good idea, but he thought the word was too easy. Emilee Elf suggested that they use a bigger word to make it more difficult. Susann Elf thought that the word "GIFTS" would be a better choice. Then Maggie Elf suggested the word "PRESENTS."



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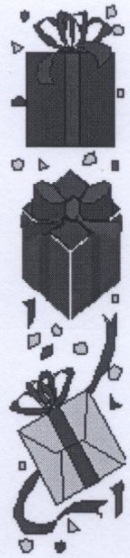

Use your mathematical skills to discover a pattern that will help Santa to solve the puzzle no matter what word the clever elves finally decide to use.

Clearly state a general rule that will tell Santa the total number of ways to trace the letters in any word. Show how you would apply your rule to the three words (toys, gifts, presents) suggested by the elves. (1 point)

General Rule.

$\boxed{2^{x-1}}$

$x \rightarrow$ number of letters in bottom row

Toys has 4 letters, so there are $2 \times 2 \times 2 = 8$ ways to trace.

Gifts - 5 letters, $2 \times 2 \times 2 \times 2 \rightarrow 16$ ways to trace.

Presents - $2^{8-1} = 2^7 = 128$ ways.

18. Locker Dilemma: You are about to enter your brand new school for the first time. The teachers, however, have gotten together and decided to perform a little ritual. All 150 students of the school need to line up and enter the school one at a time. The first student entering will open all 150 lockers. The second student will enter and close every second locker. The third student will change every third locker… and so on. You are the last in line! But as you are waiting for your turn you realize you can figure out which lockers will be open after your turn. You amaze your teachers! Which lockers are open? (1 point)

All the square numbers (numbers with an odd number of factors) will be open 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 14