Jakarta International School

8th Grade – AG1

Practice Test - BLUE

Polynomials and Quadratic Equations

Name: SOLUTIONS

Date:

Score: (–)

Polynomials Vocabulary

Write each polynomial in standard form. Then, identify each polynomial by degree AND terms.

Variable Expression	Standard Form	Identified by Degree *and* Number of Terms
$2x-5x^2+5$	$-5x^2+2x+5$	quadratic trinomial
$4x^3-3x^3$	x^3	cubic monomial
$7w-9-3w$	$4w-9$	linear binomial

Always, Never, or Sometimes

Fill in the blanks of the sentences below with ALWAYS, NEVER, or SOMETIMES.
***** If you answer SOMETIMES, include examples to prove you are correct *****

1. The sum of a trinomial and a binomial is Sometimes a monomial.

Yes: $x^2+3x+5 + (-3x-5) = x^2$

No: $x^2+3x+5 + (4x+4) = x^2+7x+9$

2. The quotient of a quadratic binomial and a linear monomial is NEVER a linear monomial.

True or False

For each statement, write true or false. Then, provide an explanation or example to demonstrate your understanding.

3. In the factoring of a trinomial, if the constant term is negative, then the signs in both binomial factors will always be negative. FALSE

$x^2+2x-3 = (x+3)(x-1)$

4. The expression $(a+b)^3$ means that the polynomial $(a+b)$ is to be used three times as a factor. TRUE $(a+b)^3 = (a+b)(a+b)(a+b)$

5. The graph of $y=(4-x)(3-x)$ opens downwards. FALSE

False $y = 12-7x+x^2$ ★ The coefficient of x^2 is positive so the graph opens up.

6. $[(2a+1)x^2-(b-3)x+5]-[(b+4)x^2-4ax+5]=3x^2+5x$

x^2: $2a+1-(b+4)=3$

$2a+1-b-4=3$

$2a-b=6$

$*\ b=2a-6$

x: $-(b-3)+4a=5$

$-b+3+4a=5$

$*\ -b+4a=2$

substitute

$-(2a-6)+4a=2$

$-2a+6+4a=2$

$6+2a=2$

$\boxed{a=-2}$

since $b=2a-6$

$b=2(-2)-6$

$b=-4-6=\boxed{-10=b}$

7. The sum of a polynomial and $4x^2-3x+2$ is $5x^3+6x^2-5$. What is the polynomial?

$(\ ?\ )$

$+\quad 4x^2-3x+2$

$5x^3+6x^2\quad -5$

$(\ ?\ )=\boxed{5x^3+2x^2+3x-7}$

8. Rewrite the following expression $4(x+1)^2-3$ in the form $C+x(B+Ax)$. Then, identify A, B, and C

$4(x^2+2x+1)-3$

$4x^2+8x+4-3$

$4x^2+8x+1$

$1+x(8+4x)$

A = 4

B = 8

C = 1

9. What non-zero integer must be placed in the square so that the simplified product of these two binomials is a binomial?

$(8x+4)(4x+\square)$

$32x^2+8\cdot\square x+16x+4\cdot\square$

to end up with a binomial

$8\cdot\square=-16$

therefore $\square=-2$

$\boxed{-2}$

Instructions: Appropriately complete each problem. Factor polynomials completely. Solve equations. When and where appropriate, show all possible solutions. Check solutions.

10. $16h^4-8h^2+1=0$

$(4h^2-1)(4h^2-1)=0$

$4h^2-1=0$

$4h^2=1$

$h^2=\frac{1}{4}$

$\boxed{h=\pm\frac{1}{2}}$

11. $3x^2-36=3x$

$-3x\quad -3x$

$3x^2-3x-36=0$

$3(x^2-x-12)=0$

$3(x-4)(x+3)=0$

$\boxed{x=4 \text{ or } -3}$

12. $x^4 + 1024$

(hint: write an equivalent expression for which you can take advantage of the difference of two squares pattern)

$= x^4 + 64x^2 + 1024 - 64x^2$

$= (x^2+32)^2 - 64x^2$

$= \boxed{(x^2+32+8x)(x^2+32-8x)}$

13. $\frac{1}{x^2} - \frac{11}{4x} + \frac{7}{4} = 0$

$\left(\frac{1}{x} - 1\right)\left(\frac{1}{x} - \frac{7}{4}\right) = 0$

$\frac{1}{x} - 1 = 0$ OR $\frac{1}{x} - \frac{7}{4} = 0$

$\frac{1}{x} = 1$ OR $\frac{1}{x} = \frac{7}{4}$

$\boxed{x = 1 \text{ OR } x = \frac{4}{7}}$

14. $\frac{2x+10}{14} = \frac{9}{3x+3}$

$(2x+10)(3x+3) = 9 \cdot 14$

$6x^2 + 36x + 30 = 126$

$6x^2 + 36x - 96 = 0$

$6(x^2 + 6x - 16) = 0$

$6(x+8)(x-2) = 0$

$\boxed{x = -8 \text{ or } 2}$

15. $-x^2 - 4x + 2 = 0$

Easiest to use the quadratic formula.

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{4 \pm \sqrt{16 - -8}}{-2}$

$x = \frac{4 \pm \sqrt{24}}{-2}$

$x = \frac{4 \pm 2\sqrt{6}}{-2} = \boxed{-2+\sqrt{6} \text{ OR } -2-\sqrt{6}}$

Check:

$-(-2+\sqrt{6})^2 - 4(-2+\sqrt{6}) + 2 = 0$

$-4 + 4\sqrt{6} - 6 - -8 - 4\sqrt{6} + 2 = 0$

$-2 + 2 = 0$ ✓

16. Your dance class has decided to perform *The Nutcracker*. *The Nutcracker* is one of the most popular Christmas holiday ballets today.

Your class chooses two primary dancers for the lead roles, one male and one female. One of the male dancer's leaps can be modeled by the equation $h = 2t - t^2$ where h is the height in feet and t is the time in seconds. One of the female dancers' leaps can be modeled by the equation $h = 3t - t^2$.

A. Sketch the graphs of the equations for the male and female dancers.

MALE

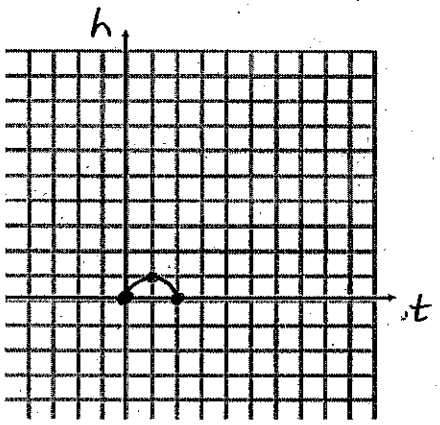
$h = t(2-t)$

x-intercepts at $t=0$ and 2

midpoint at $t=1$

so $h = 2 \cdot 1 - 1^2$

$h = 1$



FEMALE

$h = t(3-t)$

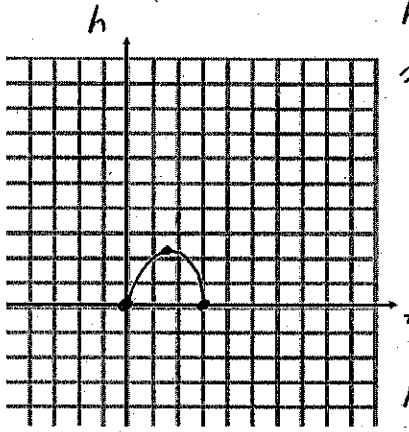
x-intercepts at $t=0$ and 3

midpoint at $t = 1.5$

$h = 3(1.5) - 1.5^2$

$h = 4.5 - 2.25$

$h = 2.25$



B. What is the maximum height reached by the male dancer when he leaps?

1 foot

C. How many seconds does it take the male dancer to reach his maximum jump height?

1 second

D. What is the maximum height reached by the female dancer when she leaps?

2.25 feet

E. How many seconds does it take the female dancer to reach her maximum height?

1.5 seconds

Use quadratic equations to solve problems 17 - 21.
Show all steps of your thinking. **Circle your answer.**

17. If the second of three consecutive positive integers is added to the product of the first and third, the result is 109. Find the integers.

Let x = the first integer

$(x+1) + x(x+2) = 109$

$x + 1 + x^2 + 2x = 109$

$x^2 + 3x - 108 = 0$

$(x-9)(x+12) = 0$

$x = 9$ or -12 BUT the integers are positive... so $x = 9$

The 3 integers are:

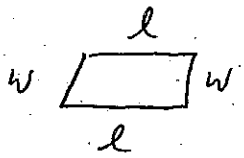
9, 10, and 11

check. $10 + 9 \cdot 11 = 109$

$10 + 99 = 109$

$109 = 109$ ✓

18. The perimeter of a rectangle is 18 meters and its area if 20 square meters. Find the length and width of this rectangle.



$2l + 2w = 18$ ← Perimeter

$2l = 18 - 2w$

$l = 9 - w$

and $l \cdot w = 20$ ← Area

$(9-w)w = 20$

$9w - w^2 = 20$

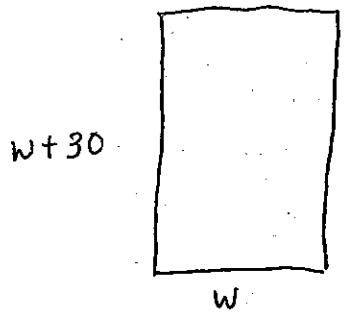
$w^2 - 9w + 20 = 0$

$(w-5)(w-4) = 0$

so $w = 5$ or 4, then $l = 5$ or 4

So, the length and width of this rectangle are 5 and 4

19. The floor of a large parking garage has an area of 400 square meters. The garage is 30 meters longer than it is wide. If each car is 2 meters long by 1 meter wide, how many lines of cars, squeezed into the garage from bumper to bumper, will fit in the parking garage? How many cars will be in each row?



$w(w+30) = 400$

$w^2 + 30w = 400$

$w^2 + 30w - 400 = 0$

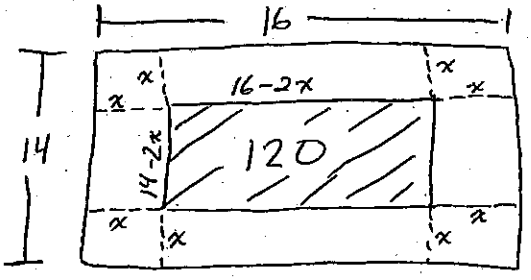
$(w+40)(w-10) = 0$

$w = -40$ or 10

a negative width is not possible, so $w = 10$ and the length is 40.

Since the cars are 2 meters long by 1 meter wide, you could either fit 10 rows with 20 cars in each row or 40 rows with 5 cars in each row.

20. Jack plans to make a pan by cutting squares from each corner of a 14 cm by 16 cm sheet of tin. After he cuts the squares from each corner, he will fold up the sides of the remaining piece. The bottom of the pan is to have an area of 120 square cm. What is the length of the side of each square that Jack should cut out?



$(16-2x)(14-2x)=120$

$224-60x+4x^2=120$

$4x^2-60x+224=120$

$4x^2-60x+104=0$

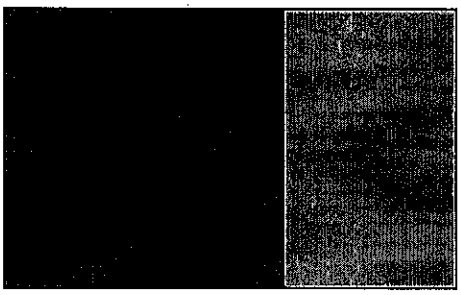
$4(x^2-15x+26)=0$

$4(x-13)(x-2)=0$

$x=13$ or 2

13 is impossible so the length of each side is 2

21. I have an interesting rectangle here. If I cut a square off it, the remaining rectangle is similar to the original one (same shape, different size).



If the width of my rectangle is one meter, what is its length? Provide an exact answer.

* The width of the original rectangle is one meter. Let the length be x.
* For the new rectangle, the length = 1 because it is the same as the width of the original rectangle. Let the new width be y.

Similar rectangles have the same ratio of length to width, so with these rectangles $\frac{x}{1}=\frac{1}{y}$

We know $y=x-1$ so...

$\frac{x}{1}=\frac{1}{x-1}$

$x(x-1)=1$ →★

→★ $x^2-x=1$

$x^2-x-1=0$

I can't factor this, so use the quadratic formula.

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{1\pm\sqrt{1--4}}{2}$

$=\frac{1\pm\sqrt{5}}{2}$

Length cannot be negative, so the length is

★ $\frac{1+\sqrt{5}}{2}$ ★