



Goal 5: Solve problems using visualization and geometric modeling

Section 1: Points, Lines, and Planes

Read the following statements and indicate if each of the following is ALWAYS TRUE (A), SOMETIMES TRUE (S) or NEVER TRUE (N). (4 points)

Justify your answer either by a written explanation or a drawing to show your understanding.

#	Statement	A/S/N	Drawing or Explanation
1.	\overline{AC} and \overline{CD} are different lines	S	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>TRUE</p> </div> <div style="text-align: center;"> <p>NOT TRUE</p> </div> </div>
2.	If l is a line parallel to the plane A and B is a plane containing the line l , then planes A and B are parallel	S	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>TRUE</p> </div> <div style="text-align: center;"> <p>NOT TRUE</p> </div> </div>

3. How many lines would be formed by n non-collinear points? Clearly explain or show how you arrive at your answer. (2 points)

<p>Lines () Regions</p> <p> (1) 2</p> <p> (2) 4</p> <p> (3) 7</p>	<p> (4) → 11 Regions</p>	<p>$1+2+3+4+\dots+n = \frac{n(n+1)}{2}$</p> <p>So, with n lines there will be $\frac{n(n+1)}{2} + 1$ Region</p>
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When you add a new line to previous lines, you will have $n+1$ more regions, So with n lines, there will be $1+1+2+3+4+\dots+n$ regions

4. Who cut the cheese? The soldier Cut the Cheese! (3 points)

A soldier cut a brick of cheese to share with his platoon. How many pieces of cheese did the soldier produce with six plane cuts?



With 0 planes, there is 1 piece.
 With 1 plane, there are 2 pieces.
 With 2 planes (perpendicular), there are 4 pieces.
 With 3 planes, there are 8 pieces.
 The 4th plane will hit the 3 planes in 1 line each, so that the cheese is cut into the number of pieces that 3 lines will create, which is 7.
 The number of pieces is now the previous number plus 7, which is 15.
 The number of pieces with 5 plane cuts will be $15 + 11 = 26$.
 The number of pieces with 6 plane cuts will be $26 + 16 = 42$ pieces

QUESTIONS 5 - 8 concern your understanding of SPHERICAL GEOMETRY

5. For each property listed from plane Euclidean Geometry, write a corresponding statement for spherical geometry. (2 points each = 6 points)

a. The shortest path between two points is a straight line segment.

The shortest path between 2 points is an arc of a great circle.

b. Two lines intersecting to form four right angles are perpendicular.

2 great circles intersecting to form 8 right angles are perpendicular.

c. Through any two points in a plane, there is a unique and infinite straight line.

Through any 2 points in a sphere, there is a unique and finite great circle

6. Compare the distance between any pole point and its equator to the length of a great circle on the same sphere. (2 points)

The distance between a pole point and its equator is $\frac{1}{4}$ the length of a great circle on the same sphere.

7. Compare and contrast lines in plane Euclidean Geometry with great circles in spherical geometry. Consider the number and type of regions created when the "line" divides the "plane." Also, consider the number of intersection points with other "lines." (2 points)

First, a line divides a plane into 2 infinite regions while a great circle divides a sphere into 2 finite regions. 2nd, two lines intersect in a single point while 2 great circles intersect in 2 points.

8. Is it possible for parallel great circles to exist? Explain. (2 points)

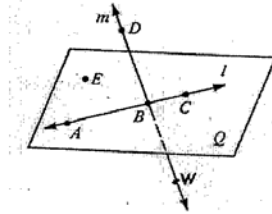
No. It is impossible to draw 2 non-intersecting great circles. 2 great circles will always intersect in 2 points

Distance, Line Segments, and Rays

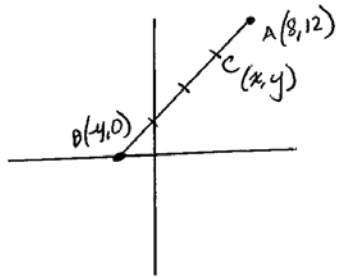
9. Do the two figures named intersect? If so, what is the intersection? (2 points)

\overline{AB} and \overline{CB} ? \overline{AC}

\overrightarrow{DB} and \overrightarrow{BW} ? \overrightarrow{DB} or \overrightarrow{DW}



10. Point C lies on \overline{AB} such that $AC = \frac{1}{4}AB$. If the endpoints of \overline{AB} are $A(8,12)$ and $B(-4,0)$, find the coordinates of C. (2 points)



$$y = \left(\frac{12-0}{4}\right) \cdot 3 = 9$$

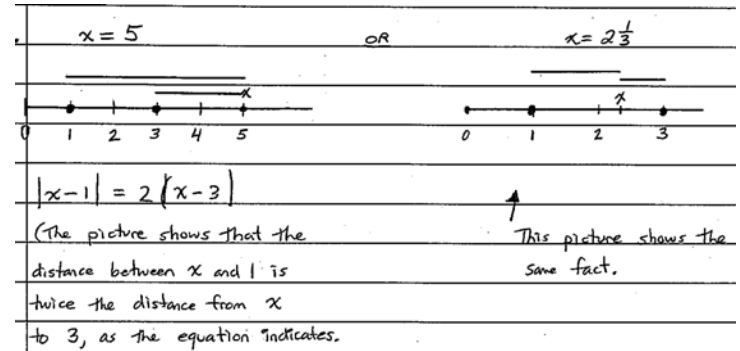
$$x = \left(\frac{8-(-4)}{4}\right) \cdot 3 + (-4) = 5$$

$(5, 9)$

11. 2 points, A and B, are on a number line. Write an expression that represents the distance between the two points. (1 point)

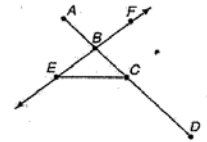
$B-A$ or $|B-A|$ or $|A-B|$

12. Find the value(s) of x satisfying the equation $|x-1| = 2|x-3|$. Draw a number line that illustrates why your answer makes sense. (3 points)



Midpoints

13. In the figure below, \overline{EC} bisects \overline{AD} at C, and \overline{EF} bisects \overline{AC} at B. Find the value of x and the measure of the indicated segment. (2 points)



$$AD = 12x - 10, AC = 3 - 2x; \overline{BC}$$

$$2AC = AD$$

$$\text{so } 2(3 - 2x) = 12x - 10$$

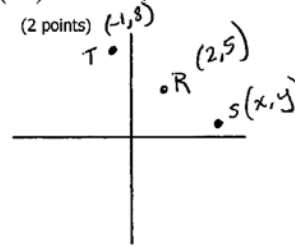
$$6 - 4x = 12x - 10$$

$$16 = 16x \quad \boxed{x=1}$$

$$\text{so } AC = 3 - 2(1) = 1$$

$$BC = \frac{1}{2}AC \text{ so } \boxed{BC = \frac{1}{2}}$$

14. If $R(2,5)$ is the midpoint of \overline{ST} and the coordinates of T are $(-1,8)$, find the coordinates of S. (2 points)



$$\left(\frac{x+(-1)}{2} = 2 \text{ and } \frac{y+8}{2} = 5 \right)$$

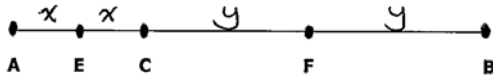
$$x = 5 \text{ and } y = 2$$

S is $(5, 2)$

15. If the endpoints of a line segment are at a and b on a number line, write an expression for the midpoint of the segment in terms of a and b. (1 point)

$$\text{midpoint} = \frac{a+b}{2}$$

16. In the figure below, C is any point between A and B, E is the midpoint of \overline{AC} , and F is the midpoint of \overline{CB} . Write a ratio comparing AB to EF. (2 points)

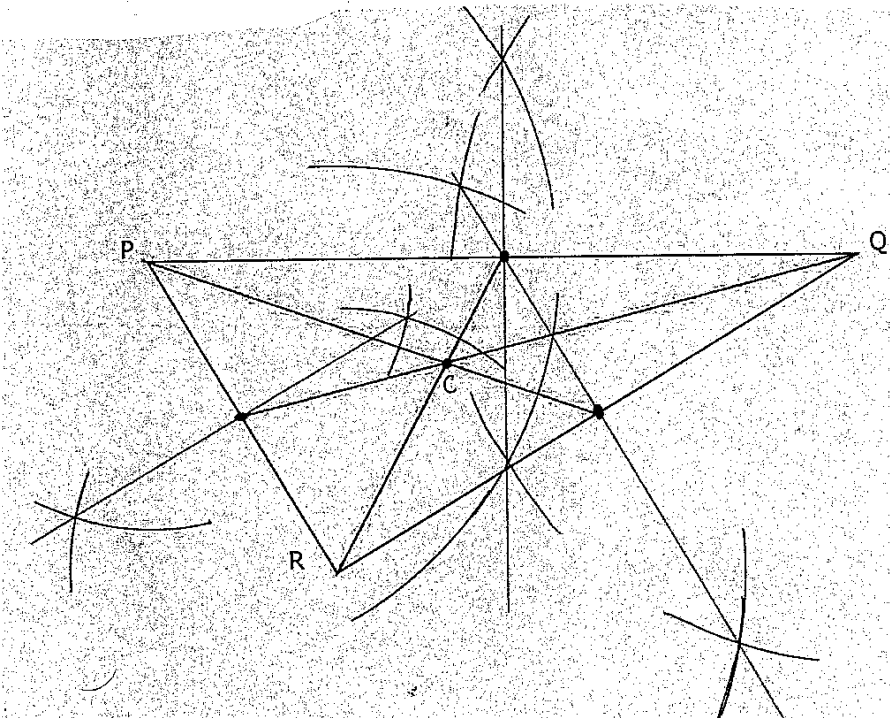


$$\frac{AB}{EF} = \frac{2x + 2y}{x + y} = \frac{2}{1} = \boxed{2:1}$$

Constructions: Complete the following construction problems.

17. **Center of Mass** (2 points)

An object's center of mass is the point where the object balances in all directions. A triangle's center of mass is located at the intersection of three line segments- the line segments connecting each of the triangle's vertices with the midpoints of the triangle segments opposite the vertices. Use a compass and straight edge to locate the following triangle's center of mass. Label the center of mass, C. (3 points)

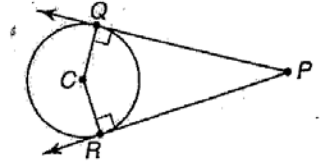


18. **Oops! Broken Glass Top** (2 points)

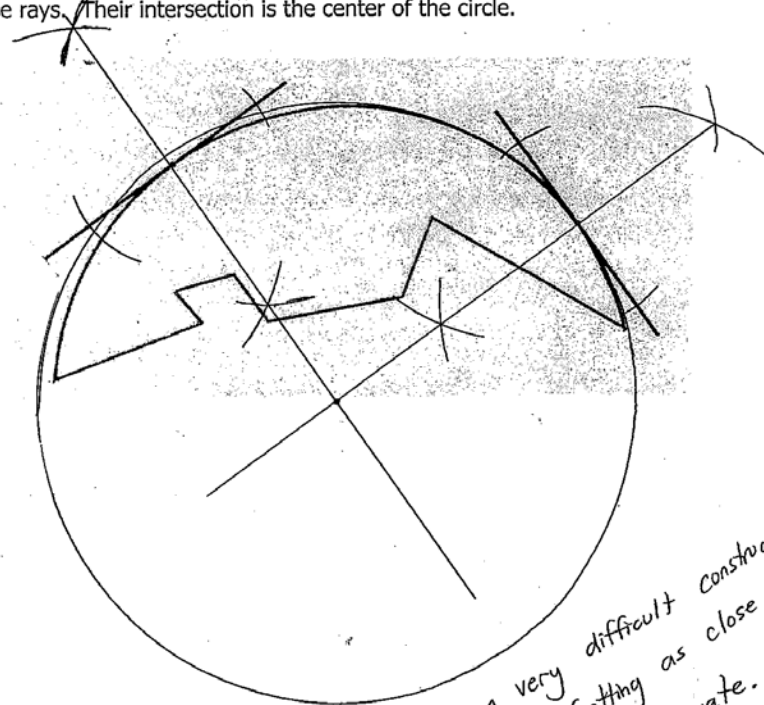
The circular glass top of your neighbor's coffee table breaks. Your neighbor is very upset and would like to replace the glass top but does not know exactly how big it was. He brings you a piece of broken glass that contains part of the boundary of the original top. He needs you to figure out how large the original glass was.

Use your compass and a straight edge to locate the center of the glass top. Then, use your compass to draw the whole circular glass top.

Hint: The center of a circle can be located by finding the intersection of two line segments which are perpendicular to lines that are tangent to the circle.



tangent- A line is tangent to a circle when it intersects the circle in exactly one point. In the figure below, \overline{PQ} and \overline{PR} are tangent to circle C. \overline{CQ} and \overline{CR} are perpendicular to those rays. Their intersection is the center of the circle.



★ This is a very difficult construction to do perfectly. Getting as close as I did above is adequate. ★