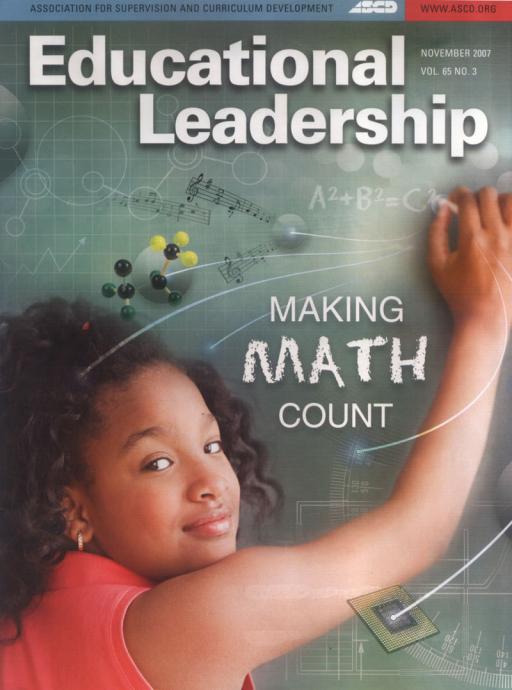


# Educational Leadership

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MAKING  
**MATH**  
COUNT

# When Students

*Through tiered instruction, students at different ends of the ability spectrum find success in math class.*

## David Suarez

I used to wonder why, despite my enthusiasm for teaching and my students' genuine interest in learning, I was missing the mark with so many middle school math students. However, after I put together Vygotsky's (1986) concept of the zone of proximal development with Mihaly Csikszentmihalyi's perspective on how to create joyful concentration or "flow" in learning, I understood my classroom dynamics better. According to Csikszentmihalyi (1990), enjoyment in learning "appears at the boundary between boredom and anxiety, when the challenges are just balanced with a person's capacity to act" (p. 52). Recognizing this truth, I couldn't ignore the obvious. My underperforming students were either bored or overwhelmed.

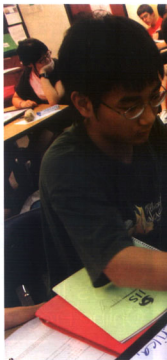
When I began teaching integrated algebra-geometry classes to 8th graders at Jakarta International School in Indonesia, I decided to structure my classroom so that students could choose their own zone of proximal development, the learning task that is just challenging enough to be motivating. Jakarta International School is a private, international K-12 institution with approximately 650 students in the middle grades (6-8). My students ranged from those who had been recommended for remedial math courses to those who had already

learned much of the upcoming year's mathematics curriculum. The question of how to give each student in this diverse group the opportunity to grow weighed on my mind.

As I struggled to select an appropriate common learning destination for a group in which the starting points were so different, I concluded that there was no single appropriate end goal. Instead, in 2005, a fellow teacher and I developed a tiered program of instruction that enables students to study the same content at different levels of challenge. We began implementing this system with 8th grade math students in the 2006-07 school year and are considering using a similar approach in 8th grade science.

## Setting Up Tiered Instruction

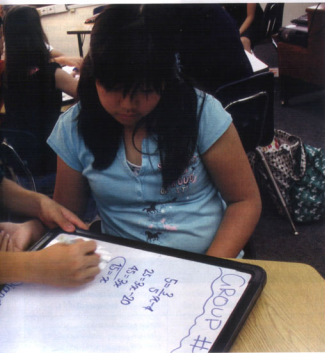
We found that the most helpful approach to tiered assessments was to organize units of study into thematic units, with specific skill outcomes designated for each theme. For example, in a thematic unit on graphing, skill outcomes included converting between graphical, numerical, and symbolic representations of data and analyzing functions and patterns. Establishing these broad units enabled us to keep the number of traditional summative assessments to a manageable level by assessing at the end of each thematic unit rather than assessing each skill separately.



Our next step was to distinguish among foundational, intermediate, and advanced levels of understanding for each math skill. We designated each level of mastery by a color. Students choose their color level for each thematic unit and have the opportunity to vary their choices from unit to unit.

Green-level tasks meet the standard for proficiency for 8th grade mathematics at Jakarta International School.

# Choose the Challenge



Blue-level tasks extend familiar skills into more complex work. To succeed at blue tasks, students must be able to recognize the subtleties that make a problem more complex and must be sharp in the required skills. Black-level challenges are the most complex and are appropriate for highly advanced and motivated students. They require the creative application and extension of skills and sometimes require students to

carry out unfamiliar tasks. Figure 1 presents examples of tasks at each level of challenge.

## The Power of Student Choice

At the outset of my algebra-geometry course, I explained the different levels of tasks and assessments and asked each student to select the level of challenge most appropriate for his or her individual learning. I encouraged students

to select the level of challenge that would help them maximize the speed and quality of their learning. Drawing on Jensen's (1998) work on how stress affects learning, I explained that choosing tasks that were too hard or too easy would lead to less than ideal stress levels: Tasks that were just challenging enough would make learning interesting but not overwhelming.

## My underperforming students were either bored or overwhelmed.

Consistent with Glasser's (1986) choice theory, empowering students to select their own level gives three major benefits:

- Students find choices motivating—often the key to achievement for middle schoolers.
- Students benefit from the opportunity to make decisions. Learning to reflect on personal learning and adjust tasks accordingly is a great skill for middle school students.
- Students can't conclude that a grouping decision made on their behalf is unfair or inappropriate.

My students consistently made appropriate choices and enthusiastically accepted this responsibility. One student, Ruth, wrote,

Being asked to choose from three different levels of difficulty has given me more choices and opportunities to challenge myself. . . . I feel that I have more "say" in the level of math I am learning.

### How It Works in the Classroom

A typical day in my classroom begins with a brief period of whole-class instruction intended to illuminate the lesson's essential understandings. Students then select practice assignments at the level of challenge they desire. They work on these assignments for approximately 40 minutes in class and complete them at home. Students can choose a different level every day if they want, and their methods of operating vary. Some learners like to complete the same challenge level all the time, others switch often between levels.

In-class practice is essential because it gives students the opportunity to work with others who are interested in tackling similar challenges—and gives me an opportunity to help students. Most students enjoy working with others who are attempting similar problems. There's usually a buzz in the air as students seek out others, either as partners in problem solving or as helpers if a student is stuck. I seat students at heterogeneously arranged tables but let them move

## Student performance across the grade level has increased.

around the room as needed during individual practice time.

Figure 2 shows three different levels of assignments that I provided my students after a lesson on triangle properties. This lesson was one of several in a thematic unit on angle relationships. At the unit's conclusion, students selected a summative assessment linked to angle relationships at a challenge level of their choice.

I allow students to look at all of the assessments, compare them, and make their choice: They can even start one assessment, quit, and switch to a different level test if time allows. A student who does poorly on a higher-level assessment can go back and take the green-level assessment to demonstrate at least grade-level proficiency.

### The Results

#### Higher Achievement . . .

Our teachers implementing tiered math instruction have been extremely pleased with the results so far. Students are performing at higher levels of achievement, are more motivated, and are assuming more responsibility for their learning. What has surprised us is that, even though we have used the foundational level of performance in 8th grade mathematics as the "starting" green level, a level that no student is compelled to exceed, student performance across the grade level has increased.

Students at our school seem to strive for, and achieve, an A-/B+ level of excellence no matter what assessment level they select. It follows, then, that if they are scoring at such a level on harder tests, their achievement has risen. By midway through the 2006-07 school year, teachers had asked 8th graders to make a total of 883 choices among levels of math assessments, and students most often chose harder-than-basic tests. Students selected green-level tests, which are

FIGURE 1. Sample Student Tasks at Different Challenge Levels

Lesson Topics	Green-level tasks (foundational)	Blue-level tasks (intermediate)	Black-level tasks (advanced)
Problem solving with linear equations	The difference in the ages of two people is 8 years. The older person is 3 times the age of the younger. How old is each?	The length of a rectangle is 3 less than half the width. If the perimeter is 18, find the length and width.	When asked for the time, a problem-posing professor said, "If from the present time, you subtract one-sixth of the time from now until noon tomorrow, you get exactly one-third of the time from noon until now." What time was it?
Understanding slope	Find the slope of the line passing through the following pair of points: (-4, 6) and (-3, 2).	Find a so that the line connecting the points (-2, -3) and (2, 5) is parallel to the line connecting the points (6, a) and (0, -4).	If $a > 1$ , what must be true about $b$ so that the line passing through the points $(a, b)$ and $(1, -3)$ has a negative slope?

similar to the whole-group assessments we used before introducing choice, only 33 percent of the time. They chose blue-level assessments (above the proficiency standard) 59 percent of the time, and black-level tests 8 percent of the time. Thus, students are now tackling greater challenges than in the past.

At the same time, test scores are holding steady compared with previous years, indicating that overall achievement has risen. In 2005–06, students in 8th grade algebra-geometry achieved an average score of 90 percent correct on the whole-group assessments they all took throughout the year (which were at the green level of difficulty). During the first year of tiered instruction, students in this class achieved average scores within a few points of 90 percent correct on the green-, blue-, and black-level assessments they took.

Before we launched tiered instruction, students at the beginning end of the readiness spectrum tended to bring a class's average math test scores down. Now, instead of bringing average scores down on a whole-group assessment, students at the beginning end of the readiness spectrum score on green-level assessments at levels comparable to those of students taking the harder blue- and black-level tests. I have been thrilled to see students at this "green level" improve their performance.

#### ... And Eager Learners

Students consistently showed increased motivation once I gave them choices. If students believe themselves to be below average, they will generally perform below average on an assessment designed for the entire class. On the other hand, students who believe that assessments were designed with their readiness level in mind will expect themselves to be successful. Tiered learning fuels a positive self-fulfilling prophecy.



**Students learn a lot about themselves as they grapple with questions about what is best for them and move forward with new insights.**

#### Positive Perspectives

Both students who have a history of stellar achievement in math class and students who start with more basic skills feel comfortable with the tiered system. Gabi, who tends to select green-level assignments, commented, "I like having choices because you can decide whether you are ready for a harder challenge or not." Other students are enthusiastically tackling unprecedented levels of achievement. After taking a black-level assessment, Wa-Lee exclaimed, grinning, "That was hard!" When I asked Johannes how he was feeling about an upcoming black-level test, he replied, "Excited!"

At the conclusion of each unit, we ask students to reflect on how difficult or easy the work they picked was for them. Overwhelmingly, students reported feeling appropriately challenged. On 88

percent of the written reflections that 8th graders completed, students reported feeling appropriately pushed "toward the goal of maximizing their learning." On only 7 percent of these reflections did students label the assessment that they had selected as "too simple" for them, and on only 6 percent did they label their choice "too challenging."

Offering tiered choices allows students to modify future decisions if, in hindsight, they view an assessment they have selected as too simple or too challenging. With this arrangement, one student's growth and success in math need not come at the price of another's chance for the same. In fact, a very positive classroom culture has developed. Peer pressure now wields a positive influence as students take pride in confronting challenges and at times

choosing a higher level.

My students learn a lot about themselves as they grapple with questions about what is best for them and move forward with new insights. Student responses to the prompt "How did you select your challenge level? Are you satisfied with your choice?" are revealing, Vishali noted,

[Blue-level work] is what I am comfortable with. I know I am capable of blue. I am satisfied with my choice because I learned and understood many new things. I know that if I had chosen black, then I would have been stuck in chaos.

Tarisha commented,

The three-choice color system helps improve learning because it gives you the feeling that no one is forcing you to do something that you might find too stressful. It also gives you a better idea of how to be independent and not have everything be decided for you.

Parents' reactions have also been very favorable. Parents of advanced students finally feel their children are being challenged in class, whereas the parent of a student at the other end of the readiness continuum remarked, "This is the first time my child feels successful in math." Another mother enthused,

This is making my daughter think about her learning and it gives her a chance to practice decision making. This is exactly what kids should be doing in middle school.

### A Work in Progress

My colleagues and I recognize that our efforts are a work in progress. But our tiered approach to learning and assessment is positively affecting student achievement, and students and parents both prefer it to our previous curriculum. I do have a few words of caution, encouragement, and support for schools considering a similar effort.<sup>1</sup>

First, the process of developing tiered assessments and differentiating instruc-

tion gets easier over time. If you are apprehensive, move ahead and don't paralyze yourself with worry. Second, keep your eyes open for challenging math problems. There is no shortage of foundational-level problems in traditional textbooks. We have, however, found that there is a short supply of

high-level problem-solving tasks. Supplementary challenge handouts available in the teacher support materials accompanying textbooks are good places to start. I also keep my eyes open for challenging problems in mathematics books and Web sites.<sup>2</sup>

Finally, develop a grading system that

## My "Aha!" Moment

**M. Kathleen Heid, Distinguished Professor of Curriculum and Instruction, Penn State University, University Park, Pennsylvania.**


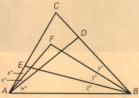
I have had several early "aha" moments in mathematics, at different times and in different contexts. As a 10-year old, I was riding with my family across the Northern Tier of Pennsylvania, returning from a family vacation trip in New England. The trip was a long one, so I occupied myself with several mental games. I would try to predict when we would reach certain landmarks on the trip, record the predictions, and then check them when we arrived. As the trip progressed, I became increasingly accurate in my predictions. Next, I decided to start a list of numbers, doubling the last one for each one I wrote. I started noticing patterns and predicting the sequence of digits in decimal places of consecutive numbers. Both times, I was surprised and pleased to notice the power in numbers (in the accuracy of prediction and in the regularity in digit patterns).

Another instance of a growing excitement about mathematics occurred the summer after 8th grade. To get ready for high school, I purchased an introductory book on physics. I was delighted to find that there were formulas that described physical action and that I could use those formulas to figure out quantities that were not given in problems. Key to excitement for young people is the ownership they can feel when their discoveries—no matter how small—are of their own making.

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FIGURE 2. Tiered Work for a Lesson on Triangle Properties

Green-level task (foundational)	Blue-level task (intermediate)	Black-level task (advanced)
<p>Find the value of <math>x</math>.</p>  <p>Source: Green-level task is from <i>Houghton Mifflin Unified Mathematics Book 1</i> by G. Rising, 1991, Atlanta: Houghton-Mifflin. Copyright 1991 by Houghton-Mifflin. Used with permission. Black-level task is from <i>Challenging Problems in Geometry</i>, by A. Posamentier and C. Salkind, 1988. New York: Dover. Copyright, 1988 by Dover. Used with permission.</p>	<p>In <math>\triangle ABC</math>, the measure of <math>\angle A</math> is three times that of <math>\angle C</math>, and the measure of <math>\angle B</math> is twice the sum of the measures of <math>\angle A</math> and <math>\angle C</math>. Find the measure of each angle.</p>	<p>In any <math>\triangle ABC</math>, <math>E</math> and <math>D</math> are interior points of <math>\overline{AC}</math> and <math>\overline{BC}</math>, respectively. <math>\overline{AF}</math> bisects <math>\angle CAD</math>, and <math>\overline{BF}</math> bisects <math>\angle CBE</math>. Prove <math>m\angle AEB + m\angle ADB = 2m\angle AFB</math>.</p> 

is compatible with the differentiation practices you implement and be transparent about it with students and parents. We experimented during our first year of tiered instruction with weighting grades based on the level of challenge the student selected. This school year, however, the Jakarta International School is attempting “standards-based reporting.” We report student performance against individual learning goals (rather than reporting one overall course grade) and report both the level of difficulty selected by the student for each learning goal (as a performance level) and the accuracy with which the student demonstrates mastery (as a letter grade). For example, a student might earn an A, B, or C on a task at any of the levels for any particular goal.

We continue to contemplate issues such as when and how much to guide students in their decision making, how to improve differentiation practices during instruction, and how to handle a situation in which the green level of

## Tiered learning fuels a positive self-fulfilling prophecy.

challenge is beyond a student’s readiness level. I’m excited at these opportunities for continued exploration into tiered instruction and assessment. The journey so far has left me feeling closer than ever before to my goal of meeting students’ needs as math learners. **■**

<sup>1</sup>For a more detailed explanation of how to implement a tiered instructional program in math, see Chapter 11 of *Making the Difference: Differentiation in International Schools* (Powell & Kusuma-Powell, 2007) or visit my blog at [www.challengebychoice.wordpress.com/](http://www.challengebychoice.wordpress.com/)

<sup>2</sup>I have found the following resources helpful as sources of higher-level problems: *Challenging Problems in Algebra* and *Challenging Problems in Geometry*, (Posamentier & Salkind, 1988). *The MathCounts School Handbooks*, available at [www.mathcounts.org](http://www.mathcounts.org).

Balanced Assessments in Mathematics, available at <http://balancedassessment.concord.org>

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